CLASS – X

SUB → MATHS

PHASE → I

CH → 18 [Trigonometrical Identities]

AIM → Using the basic trigonometric ratios and identities we will simplify some algebraic trigonometric expressions.

6) i) $\cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + \dfrac{\tan 36^\circ}{\cot 54^\circ}$

$= \cos^2 26^\circ + \cos(90^\circ - 26^\circ)\sin 26^\circ + \dfrac{\tan 36^\circ}{\cot(90^\circ - 36^\circ)}$

$\left[\begin{array}{l}\cos(90^\circ-\theta) = \sin\theta \\ \cot(90^\circ-\theta) = \tan\theta\end{array}\right]$

$= \underline{\cos^2 26^\circ + \sin^2 26^\circ} + \dfrac{\cancel{\tan 36^\circ}}{\cancel{\tan 36^\circ}}$

$= 1 + 1 = 2$

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10) ii) $\dfrac{\operatorname{cosec}^2(90^\circ-\theta) - \tan^2\theta}{2(\cos^2 48^\circ + \cos^2 42^\circ)} - \dfrac{2\tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ}{\operatorname{cosec}^2 70^\circ - \tan^2 20^\circ}$

$= \dfrac{\sec^2\theta - \tan^2\theta}{2\{\cos^2(90^\circ - 42^\circ) + \cos^2 42^\circ\}} - \dfrac{2\times\left(\frac{1}{\sqrt{3}}\right)^2 \cdot \sec^2 52^\circ \cdot \sin^2(90^\circ - 52^\circ)}{\operatorname{cosec}^2 70^\circ - \tan^2(90^\circ - 70^\circ)}$

$= \dfrac{1}{2(\sin^2 42^\circ + \cos^2 42^\circ)} - \dfrac{\frac{2}{3} \cdot \sec^2 52^\circ \cdot \cos^2 52^\circ}{\operatorname{cosec}^2 70^\circ - \cot^2 70^\circ}$

$\left[\because \operatorname{cosec}^2\theta - \cot^2\theta = 1\right]$
& $\sin^2\theta + \cos^2\theta = 1$

$= \dfrac{1}{2} - \dfrac{\frac{2}{3}\,\frac{1}{\cancel{\cos^2 52^\circ}} \times \cancel{\cos^2 52^\circ}}{1}$

$= \dfrac{3-4}{6} = -\dfrac{1}{6}$

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17) ii) L.H.S. $= \dfrac{\tan A}{\sec A - 1} + \dfrac{\tan A}{\sec A + 1} = \tan A\left(\dfrac{1}{\sec A - 1} + \dfrac{1}{\sec A + 1}\right)$

$= \tan A \left\{\dfrac{\sec A \cancel{+1} + \sec A \cancel{-1}}{(\sec A - 1)(\sec A + 1)}\right\}$

$= \dfrac{\tan A \cdot 2 \cdot \sec A}{\sec^2 A - 1}$ $\quad [(a+b)(a-b) = a^2 - b^2]$

$= \dfrac{2 \cdot \tan A \cdot \sec A}{\tan^2 A}$ $\quad [\because \sec^2 A - \tan^2 A = 1]$

$= \dfrac{\frac{2}{\cancel{\cos A}}}{\frac{\sin A}{\cancel{\cos A}}} = 2 \cdot \operatorname{cosec}^2 A = $ R.H.S. [Proved]

23) ii) L.H.S. $= \sqrt{\frac{1-\cos A}{1+\cos A}} = \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}}$ [Multiplying both Nr and Dr by $(1-\cos A)$]

$= \sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}} = \frac{1-\cos A}{\sin A}$ [$\because \sin^2 A + \cos^2 A = 1$]

$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$

$= \operatorname{cosec} A - \cot A$ = R.H.S. [Proved]

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30) ii) L.H.S. $= (\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2$

$= \sin^2 A + 2.\sin A.\sec A + \sec^2 A + \cos^2 A + 2.\cos A.\operatorname{cosec} A + \operatorname{cosec}^2 A$

$= 1 + \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} + \frac{2.\sin A}{\cos A} + \frac{2.\cos A}{\sin A}$

$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A} + 2.\left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}\right)$

$= 1 + \frac{1}{\sin^2 A \cos^2 A} + \frac{2}{\sin A \cos A}$ [$\because \sin^2 A + \cos^2 A = 1$]

$= 1 + \sec^2 A.\operatorname{cosec}^2 A + 2.\operatorname{cosec} A.\sec A$

$= (1 + \sec A \operatorname{cosec} A)^2$ = R.H.S. [Proved]

Ch. Test

12) $\cot\theta + \cos\theta = m$

$\cot\theta - \cos\theta = n$

$2\cot\theta = m+n$

$\cot\theta = \frac{m+n}{2}$

$\therefore \cos\theta = m - \cot\theta = m - \frac{m+n}{2} = \frac{2m-m-n}{2} = \frac{m-n}{2}$

$\therefore \tan\theta = \frac{2}{m+n}$, $\sec\theta = \frac{2}{m-n}$

$\because \sec^2\theta - \tan^2\theta = 1$

$\frac{4}{(m-n)^2} - \frac{4}{(m+n)^2} = 1$

or, $4\left[\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right] = 1$

or, $4\left[\frac{4mn}{(m+n)^2(m-n)^2}\right] = 1$

or, $16mn = (m+n)^2(m-n)^2$

$= (m^2 - n^2)^2$ [Proved]

[H/W: → Ch 18 → 33, 34, 35
Ch. Test → 11, 16]